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Data Models and Query Languages
Summerterm 2013

2. Exercise Sheet: Chase & Datalog

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Exercise 1 (Constraints in First-Order Logic)
Consider the following database schema.

$\texttt{hasAirport}(c\_id)$
$\texttt{fly}(c\_id1, c\_id2, dist)$
$\texttt{rail}(c\_id1, c\_id2, dist)$

Specify the constraints below in First-order Logic and indicate if your specification is a tuple-generating dependency, equality-generating dependency, or none of both. In case of tuple-generating or equality-generating dependencies additionally give their *body* and *head*.

a) α_1: If a city has an airport, then there is at least one flight departing from this city.
b) α_2: The distance of a rail connection functionally depends on the departure and destination station, i.e. there is only one unique distance for every rail connection.
c) α_3: There is at least one flight and one train connection listed in the database.
d) α_4: Starting from Frankfurt, all cities with an airport can be reached either by direct flight or by a flight with only one intermediate stop.
e) α_5: All pairs of cities with an airport that have a direct train connection also have a direct flight connection.

Exercise 2 (Chase Application)
Consider the schema from Exercise 1, the constraint set $\Sigma := \{\alpha_1, \alpha_2, \alpha_3\}$ with

$$\alpha_1 := \forall\, c_1, c_2, c_3, d_1, d_2\ (\texttt{rail}(c_1, c_2, d_1), \texttt{rail}(c_2, c_3, d_2) \rightarrow \exists\, d_3\ \texttt{rail}(c_1, c_3, d_3))$$
$$\alpha_2 := \forall\, c_1, c_2, d_1, d_2\ (\texttt{fly}(c_1, c_2, d_1) \wedge \texttt{fly}(c_2, c_1, d_2) \rightarrow d_1 = d_2)$$
$$\alpha_3 := \forall\, c_1, c_2, d_1\ (\texttt{fly}(c_1, c_2, d_1) \rightarrow \exists\, d_2\ \texttt{fly}(c_2, c_1, d_2))$$

and the Conjunctive Query

$$Q: \quad \texttt{ans}(C_3) \leftarrow \texttt{rail}(\mathit{Freiburg}, C_1, D_1), \texttt{rail}(C_1, C_2, D_2), \texttt{fly}(C_2, C_3, D_3).$$

a) Describe the semantics of the constraints and the query informally.
b) Which constraints from Σ are satisfied by $body(Q)$? Does $body(Q)$ satsify Σ?
c) Chase query Q with Σ. Provide all intermediate results (= chase steps). Does it hold that $body(Q^{\Sigma}) \models \Sigma$?

Exercise 3 (Datalog, Transitive Closure)
Given a directed graph G with edge relation $E(a, b)$, which means there is an edge from a to b in G.

a) Give three different Datalog^{+} programs which compute the transitive closure of G.
b) Let k be the length of the longest path in G. Determine the number of iterations which is needed for each version to compute the transitive closure.
c) Apply the naive algorithm evaluation of the three programs on the database:

$$E(1,2), E(2,3), E(3,4), E(4,5)$$

Exercise 4 (Datalog, Equivalence)
Consider the following two Datalog^{+} programs

$$\sqcap_1: \quad Buys(X,Y) \leftarrow Likes(X,Y)$$
$$\quad\quad Buys(X,Y) \leftarrow Knows(X,Z), Buys(Z,Y)$$

$$\sqcap_2: \quad Buys(X,Y) \leftarrow Likes(X,Y)$$
$$\quad\quad Buys(X,Y) \leftarrow Knows(X,Z), Likes(Z,Y)$$

Proove or falsify that $\sqcap_1 \equiv \sqcap_2$.

Exercise 5 (Datalog, Shortest Paths)
Given a directed **acyclic** graph G with edge relation $E(a, b)$, which means there is an edge from a to b in G. Give a stratified Datalog program which computes the shortest paths in G wrt. the number of edges.

Exercise 6 (Datalog, 3-Coloring of Graphs)
Let $G = (V, E)$ be a graph with $V = \{1, 2, 3, 4, 5\}$ and $E = \{(1, 2), (1, 4), (1, 5), (2, 3), (2, 4), (3, 4), (4, 5)\}$.

Consider the following Datalog^{+} program

$$\sqcap: \quad Color(N, blue) \leftarrow V(N), not\ Color(N, green), not\ Color(N, red)$$
$$\quad Color(N, green) \leftarrow V(N), not\ Color(N, blue), not\ Color(N, red)$$
$$\quad Color(N, red) \leftarrow V(N), not\ Color(N, green), not\ Color(N, blue)$$
$$\quad NonColoring(N) \leftarrow E(N, M), Color(N, C), Color(M, C)$$

Give a stable model of $\sqcap$ where *NonColoring* is empty, i.e. demonstrate that the given graph is 3-colorable.